

## Electron collisionless layers near evaporating plasma-heated anodes

Russell G. Keanini\*

*Department of Mechanical Engineering and Engineering Science, University of North Carolina at Charlotte, Charlotte, North Carolina, 28223*

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Electron collisionless (Knudsen) layers near evaporating plasma-heated anodes are analyzed, and a corresponding general model is formulated. Since the ratio of ion to electron mean free paths is small, the layer is subdivided into ion continuum and ion collisionless layers. Distribution functions for charged species are derived and auxiliary non-problem-specific relationships that are required to complete the model are given. The principal results are as follows: (i) the electron distribution function is little affected by evaporation;  $f_e$  is essentially identical to the distribution found near nonevaporating anodes. (ii) Neutral evaporating particles are heated to the plasma heavy particle temperature well within the electron Knudsen layer. (iii) An analytical expression for the heavy particle temperature distribution across the electron Knudsen layer is derived.

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Plasma sheaths form near solid and liquid boundaries that are subjected to plasma heating. Due to an unbalanced space charge, relatively large electric fields can exist within the sheath, significantly influencing the energy flux to the anode. Although significant anode evaporation can occur, particularly at high energy densities, evaporation effects on the near-anode region have not received thorough theoretical attention (see, e.g., [1–7]) and presently lie outside the realm of direct measurement [8]. This paper's objectives are to first develop a realistic physical picture of the near-anode region, taking into account anode vaporization, and then to formulate a self-consistent mathematical model of the region. The hybrid model developed is applicable to the electron collisionless (Knudsen) layer, and in contrast with earlier formulations [2,4], accounts for heavy particle collisions and energy transfer within this layer. The model applies to anode heating by steady, high intensity (current  $\geq 50$  A), atmospheric pressure plasmas, assumes that the anode liquid is at or near its boiling point, and allows for any number of singly and multi-ionized species. Thus, the model is relevant to a number of plasma-based materials processing applications, e.g., gas tungsten arc welding.

In order to develop the model, we first consider the relative magnitudes of the Debye shielding length  $\lambda_D$  and the electron mean free path  $\lambda_e$ . In particular, when  $\lambda_e/\lambda_D \ll 1$  the sheath can be modeled as a continuum, whereas a kinetic formulation is required for  $\lambda_e/\lambda_D \gg 1$ . Preliminary estimates of  $\lambda_e$  and  $\lambda_D$  follow by considering an atmospheric argon plasma over the approximate range of electron temperatures expected near a *nonevaporating* anode [2]:  $8000 \text{ K} \leq T_e \leq 12000 \text{ K}$ .  $\lambda_e$  can be estimated using  $\lambda_e = \bar{C}_e/\nu_e$ , while  $\lambda_D = (\epsilon_0 k T_e / n_0 e^2)^{1/2}$ . In these expressions,  $\epsilon_0$  is the permittivity constant,  $k$  is Boltzmann's constant,  $n_0 = n_i = n_e$  is the charged particle

number density within the bulk plasma,  $T_e$  is the electron temperature,  $\bar{C}_e$  is the mean electron speed, and  $\nu_e$  is the average electron momentum-transfer collision frequency. Considering the last quantity first,  $\nu_e = \sum_{s=1}^N \bar{C}_e \bar{Q}_{es} n_s (m_e/m_{es})^{1/2}$ , where  $\bar{Q}_{es}$  is the energy averaged cross section for collisions between electrons and species of type  $s$ ,  $n_s$  is the number density of  $s$ ,  $m_e$  is the electron mass, and  $m_{es} = m_e m_s / (m_e + m_s)$  is the reduced mass. At  $T_e = 12000 \text{ K}$ , energy averaged cross sections for collisions between electrons and argon atoms are on the order of  $10^{-20} \text{ m}^2$  [9]. Employing the relation [9]  $\bar{Q}_{ei} = 6\pi b_0^2 (\ln \Lambda - 1.37)$ , where the collision and plasma parameters are given by  $b_0 = Ze^2 / (12\pi k T_e)$  and  $\Lambda = \lambda_D / b_0$ , we find that energy averaged cross sections for collisions between  $\text{Ar}^{+1}$  ions and electrons,  $\bar{Q}_{ei}$ , and between electrons,  $\bar{Q}_{ee}$ , are both on the order of  $10^{-17} \text{ m}^2$ . Thus, since number densities for charged and neutral species within the bulk plasma are approximately equal ( $n_e = n_i \approx n_n \approx 10^{23} \text{ m}^{-3}$ , as estimated by the Saha equation), charged particle collisions predominate and  $\lambda_e \approx 10^{-6} \text{ m}$ . Note that this estimate is consistent with the estimate reported by Dinulescu and Pfender [1]. At  $T_e = 8000 \text{ K}$ ,  $n_e \approx 1 \times 10^{22} \text{ m}^{-3}$  [2],  $n_n \approx 10^{24} \text{ m}^{-3}$  [3], and electron-neutral species collisions become as prominent as electron-electron collisions; in this case,  $\lambda_e \approx 1 \times 10^{-5} \text{ m}$ . Considering the associated Debye lengths,  $\lambda_D$  is on the order of  $10^{-8} \text{ m}$  when  $n_0 = 10^{23} \text{ m}^{-3}$  and  $T_e = 12000 \text{ K}$ , and on the order of  $10^{-7} \text{ m}$  when  $T_e = 8000 \text{ K}$  and  $n_0 = 1 \times 10^{22} \text{ m}^{-3}$ . Thus, since the ratio  $\lambda_e/\lambda_D$  is on the order of  $10^2$  over the range of electron temperatures and densities expected near nonevaporating anodes, the sheath in this case lies well within the electron collisionless layer.

We can show that our preliminary  $\lambda_e$  and  $\lambda_D$  estimates remain valid near evaporating anodes. To prove this, we first consider the average number of collisions  $n_{ah}$  required to heat evaporating particles (having temperature  $T_a$ ) to the plasma's heavy particle temperature  $T_h$ . The quantity  $n_{ah}$  is given by  $n_{ah} = 3k(T_h - T_a) / (2\Delta \bar{\epsilon}_{ah})$

\*FAX: (704) 547-2352. Electronic address: rkeanini@unccsun.uncc.edu

$= (m_a + m_h)/(2m_{ah})$ , where  $\Delta \bar{\epsilon}_{ah}$  is the average energy transferred per elastic collision between heavy and evaporate particles [9]. The corresponding thermalization time scale is  $\tau_{ah} = n_{ah}/\nu_{ah}^{(1)}$ , where  $\nu_{ah}^{(1)} = g_{ah} Q_{ah}^{(1)} n_h$  is the momentum transfer collision frequency and  $g_{ah}$  is the relative speed between heavy and evaporate particles. A conservative estimate of  $\tau_{ah}$  follows by assuming a relatively small cross-section of  $Q_{ah}^{(1)} = 10^{-19} \text{ m}^2$  [9], and by setting  $T_h = 5000 \text{ K}$  [1] and  $n_h (= n_i + n_n) = 1.4 \times 10^{24} \text{ m}^{-3}$  [3]. Approximating  $g_{ah}$  as the heavy particle thermal speed within the plasma ( $\approx 1600 \text{ m/s}$ ), we obtain a *minimum* collision frequency of  $\nu_{ah}^{(1)} \approx 2 \times 10^8 \text{ s}^{-1}$ . Since heavy plasma particles and evaporate particles have masses of the same order of magnitude,  $n_{ah} \approx 1$ . Thus,  $\tau_{ah} \approx 1 \times 10^{-8} \text{ s}$ . (Due to the small mass of electrons, electron-evaporate energy transfer is much less effective, with the shortest realistic time scale for energy transfer being on the order of  $10^{-6} \text{ s}$ .) Since the speed of evaporate particles  $v_{1v}$  is on the order of  $10^2 \text{ m/s}$  [10], the *maximum* distance required to heat evaporate to  $T_h$  (given by  $v_{1v} \tau_{ah}$ ) is  $1 \times 10^{-6} \text{ m}$ . In the case of iron or aluminum alloys heated by argon plasmas, cross sections for collisions between cool Fe or Al atoms and neutral and ionized Ar and hot neutral and ionized evaporating particles are estimated to be on the order of  $10^{-18} \text{ m}^2$  or more [9]. In this case, the thermalization distance will be at least an order of magnitude smaller than the above estimate. Thus, in many practical circumstances, evaporating particles reach  $T_h$  well within the electron collisionless layer. Since evaporate and native heavy particle masses are of the same order of magnitude, electron cooling within the bulk plasma clearly does not occur, proving that  $T_e$ , and thus  $\lambda_e$  and  $\lambda_D$ , are unaffected by evaporation.

Considering now the ion mean free path, we find that over the range  $8000 \text{ K} \leq T_e \leq 12000 \text{ K}$ ,  $\lambda_i$  is two orders of magnitude smaller than  $\lambda_e$ , in agreement with Dinulescu and Pfender [1]. Thus, in analogy with the conditions determining a collision-dominated electron gas [9], ions over most of the electron Knudsen layer are collision dominated and can be considered a continuum (see also Ref. [4]). We thus model ions within the electron Knudsen layer as residing in either a continuum ion layer or in a very thin collisionless layer abutting the anode (Fig. 1).

Having established a physical picture of the electron Knudsen layer, we now develop a suitable mathematical model. Within the collisionless electron and ion layers, the following form of Boltzmann's equation applies:

$$w_1 f_{s,x_1} - eZ\phi_{,x_1} f_{s,w_1} / m_s = 0, \quad (1)$$

where partial derivatives are denoted with a comma,  $Z$  is the net number of charges on particle  $s$ ,  $m_s$  is the mass of  $s$ ,  $\phi = \phi(x_1)$  is the electric potential, and  $x_1$  is the coordinate direction normal to the anode surface. The solution to (1) is given by

$$f_s(\mathbf{x}, \mathbf{w}) = a_s \exp[b_s(m_s w_1^2 / 2 + eZ\phi)] \times \exp[-m_s(w_2^2 + w_3^2) / (2kT_s)], \quad (2)$$

where  $T_s$  is the local temperature of  $s$ ,  $w_2$  and  $w_3$  are orthogonal absolute particle velocities in planes parallel to

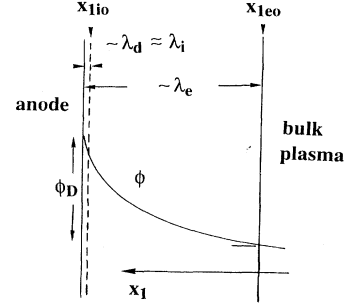


FIG. 1. Schematic of the electron Knudsen layer. Ions reside in a collision-dominated continuum over most of the layer but are collisionless immediately adjacent to the anode.

the anode, and  $w_1$  is the corresponding  $x_1$  component.

The parameters  $a_s$  and  $b_s$  in (2) are determined by taking the first two moments of the distribution function. Assuming that the anode is perfectly catalytic and that the electric field at the outer edge of the electron Knudsen layer (located at  $x_{1eo}$ ) is small, only electrons with zero or positive normal velocities ( $w_1 \geq 0$ ) pass into the electron Knudsen layer. In this case, the minimum absolute normal velocity for incident electrons is  $w_{1e}^{\min}(x_1) = \sqrt{2\phi(x_1)/m_e}$  [where, for convenience, we set  $\phi(x_{1eo}) = 0$ ]. Similarly, within the ion collisionless layer, the minimum ion velocity  $w_{1i}^{\min}(x_1) = -\sqrt{2eZ(\phi_D - \phi(x_1))/m_i}$  corresponds to ions that nearly reach the anode and then accelerate backward. Here,  $\phi_D$  is the potential at the anode.

Using the above limiting electron velocity, we obtain expressions for  $a_e$  and  $b_e$  in the electron distribution function  $f_e$ ,

$$a_e = n_{e0}/D (m_e/2\pi kT_e)^{3/2}, \quad b_e = 1/(kT_e D^2), \quad (3)$$

where  $D = [1 + \xi]^{1/2}$ ,  $\xi = \epsilon \{ 16/\pi - 4 + 32/\pi [\epsilon(1/\pi - 4) + 4]^{1/2} \}$ ,  $\epsilon = m_e v_1^2 / (kT_e)$ ,  $n_{e0} = n_e(x_{1eo})$ , and  $v_1$  is the mass average velocity in the  $x_1$  direction. Neglecting a small contribution due to electrons,  $v_1$  can be obtained using  $v_1 = (\rho_v v_{1v} + \sum_{i=1}^{N_I} \rho_i v_{1i}) / \rho$ , where  $\rho_v$  and  $v_{1v}$  are the evaporating mass density and normal velocity,  $\rho$  is the sum of all mass densities,  $N_I$  is the number of ionic species,  $v_{1i}$  is  $i$ th ionic specie's mean normal velocity, and  $\rho_i = n_i m_i$ . Since the anode material is at or near its boiling point, evaporating and native neutral number densities within the near-anode region are at least an order of magnitude larger than charged particle number densities [1,3]. Thus,  $v_1$  essentially equals  $v_{1v}$ , so that  $\xi = O[m_e v_{1v}^2 / (kT_e)] \ll 1$ . Thus, the electron distribution is virtually identical to that found near *nonevaporating* anodes [11].

Using the above minimum ion velocity within the ion collisionless zone leads to two relationships for  $a_i$  and  $b_i$ ,

$$n_{io} = a_i (2\pi^3 k^2 T_{io}^2 / b_i m_i^3)^{1/2} \times \exp(-b_i Z e \phi_o) \text{erfc}[\sqrt{b_i Z e (\phi_D - \phi_o)}] \quad (4)$$

and

$$(kT_{i0} - 1/b_i - m_i v_1^2) \operatorname{erfc}[\sqrt{b_i Z e (\phi_D - \phi_0)}] / 2 \\ + \exp[-b_i Z e (\phi_D - \phi_0)] (m_i / b_i \pi)^{1/2} \\ \times \{ [Z e (\phi_D - \phi_0) / m_i]^{1/2} + \sqrt{2} v_1 \} = 0, \quad (5)$$

where  $x_{1i0}$  is the nominal boundary of the ion collisionless zone, and where  $n_{i0} = n_i(x_{1i0})$  and  $T_{i0} = T_i(x_{1i0})$ .

Within the ion continuum zone, we assume that ions are in equilibrium with respect to the mass average velocity  $v_1$ . This assumption requires that mean ion velocities equal  $v_1$ , a condition that should be approached due to the short mean free path for momentum transfer between heavy species. The corresponding displaced Maxwellian distributions are described by

$$a_i^0 = n_i^0 (m_i / 2\pi k T_i)^{3/2}, \quad b_i^0 = 1/k T_i. \quad (6)$$

Although the problem of describing the electron Knudsen layer in the presence of evaporation is, in principle, complete, auxiliary relationships are needed to close the problem. In particular, relationships describing  $\phi = \phi(x_1)$ ,  $n_e = n_e(x_1)$ ,  $n_i = n_i(x_1)$ ,  $T_i = T_i(x_1)$ ,  $v_1 = v_1(x_1)$ ,  $\phi_D$ ,  $\phi_0$ , and  $T_e$  are needed. Maintaining a non-problem-specific formulation, we briefly outline these relationships. The potential  $\phi(x_1)$  follows from the Gauss law

$$\frac{d^2 \phi}{dx_1^2} = -\frac{e}{\epsilon_0} \left[ \sum_{i=1}^{N_i} n_i - n_e \right], \quad (7)$$

where thermionic emission has been neglected. Note that based on Freund's data [12] (which shows negligible electron impact ionization of singly ionized Fe, Al, Cu, Si, Mg, Pb, and Ga at  $T_e < 5$  eV), the density of multi-ionized anode particles is typically small. The number density of charged species is obtained by integrating corresponding distribution functions over the velocity ranges determined above. Thus, for electrons

$$n_e(x_1) = n_{e0} / 2 \exp(e\phi / k T_e D^2) \operatorname{erfc}(\sqrt{e\phi / k T_e D^2}), \quad (8)$$

while within the ion collisionless layer

$$n_i(x_1) = a_i (2\pi^3 k^2 T_i^2 / b_i m_i^3)^{1/2} \exp(-b_i Z e \phi) \\ \times \operatorname{erfc}[\sqrt{b_i Z e (\phi_D - \phi)}]. \quad (9)$$

Within the continuum ion layer, the Boltzmann relation results are

$$n_i^0(x_1) = n_{i0}^0 \exp(-e\phi / k T_i). \quad (10)$$

(Note that by setting  $D=1$  in (8), corresponding to zero evaporation, we recover the expression obtained by Sutton and Sherman [11].)

The ion temperature distribution within the ion continuum layer follows from the heavy particle energy equation [13]

$$\left(\frac{d}{dx_1}\right) k (n_h T_h v_1)_{,x_1} = (K_h T_h)_{,x_1} \quad (11)$$

where we have assumed that  $T_i(x_1) = T_h(x_1)$  and that derivatives in the  $x_1$  direction are dominant. In this equation,  $K_h$  is the heavy particle thermal conductivity [13] and  $n_h = \sum_{i=1}^{N_i} n_i + n_n$ . Note that there is no col-

lisional energy transfer from electrons to ions. Also note that condensation of evaporate at the anode occurs; a method for estimating the condensate mass flux is given by Anisimov [14]. The solution to (11) reads

$$T_h = c_0 \exp[-\zeta(x_1)] \int \exp[\zeta(x_1)] K_h^{-1}(x_1) dx_1 \\ + c_1 \exp[-\zeta(x_1)], \quad (12)$$

where  $\zeta(x_1) = \int \{ 5kn_h(x_1)v_1(x_1) / [2K_h(x_1)] \}$ , and where  $c_0$  and  $c_1$  are determined by problem-specific conditions at the boundaries to the ion continuum zone. With regard to this last point, due to the short mean free path for heavy species, one can set  $T_h$  equal to the bulk heavy particle temperature at the outer boundary and to the anode temperature  $T_a$  at the inner boundary. To complete evaluation of (12), the heavy particle number density  $n_h (= n_v + \sum_{i=1}^{N_i} n_i)$  can be determined using expressions given above and by an appropriate equation of state for the evaporate (e.g., the ideal gas law). In addition,  $v_1 \approx v_{1v}$ , which can be evaluated using  $v_1 v = K T_a^{-1/2} \exp(-B/T_a)$ , where  $K$  and  $B$  are material-specific parameters [15]. Finally, the heavy particle thermal conductivity can be determined via the Chapman-Enskog approach [16].

The overall potential increase  $\phi_D$  is obtained by calculating the total current density within the ion collisionless zone. In the case where two singly ionized species are present (i.e., ionized plasma gas and ionized anode atoms), the result is

$$J_0/e = a_e (2\pi k T_e / b_e m_e^2) \\ - \sum_{j=1}^2 a_j (2\pi k T_i / b_j m_j^2) \exp(-b_j e \phi_D). \quad (13)$$

Notice that multi-ionized species can be readily accommodated in the calculation. The fact that every integral is independent of  $x_1$  provides a useful check on the consistency of the distribution function derivations given above. The boundary of the ion collisionless zone,  $x_1 = x_{1i0}$ , and the corresponding potential,  $\phi_0 = \phi(x_{1i0})$ , are obtained in the course of solving the Gauss equation [2]. Similarly, the nominal boundary to the electron Knudsen layer,  $x_{1e0}$ , determined by solving the bulk plasma problem, corresponds to the location where spatial variations in  $n_e$  are comparable to the local magnitude of  $n_e$  [2]. Note that electric potential variations over distances greater than  $\lambda_e$  can be readily accommodated by replacing  $\phi$  in all previous expressions with  $\phi - \phi(x_{1e0})$  [where  $\phi(x_{1e0})$  is determined when  $x_{1e0}$  is determined]. The remaining parameters in the problem,  $n_{i0}^0 = n_i^0(x_1 = x_{1e0})$ ,  $n_{e0} = n_e(x_1 = x_{1e0})$ ,  $T_{i0} = T_h(x_1 = x_{1i0})$ ,  $T_e = T_e(x_1 = x_{1e0})$ , and  $J_0$ , are problem specific and are determined by conditions outside the electron Knudsen layer. Based on the preceding discussion,  $T_{i0}$  can in many cases be taken as  $T_a$ .

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